

Error Propagation

SCIENTIFIC NOTATION

- Form: $a \times 10^b$, $\{-10 < a < 10\}$
 - Ex: $0.000000625 = 6.25 \times 10^{-7}$
 - Ex: $894000000 = 8.94 \times 10^8$
- Multiplying/Dividing
 - $(a \times 10^b)(c \times 10^d) = ac \times 10^{b+d}$
 - Ex: $(1.8 \times 10^2)(2.0 \times 10^3) = 3.6 \times 10^5$
- Adding/Subtracting
 - Can only be done (easily) if b of both numbers is the same
 - Ex) $(3.4 \times 10^4) + (2.1 \times 10^4) = 5.5 \times 10^4$

SIGNIFICANT FIGURES

- All measurements have some degree of uncertainty
- Sig figs include all certain digits, and one uncertain digit
- Significant digits include:
 - All digits that are not 0
 - 0 between two non-zero digits
 - Ex: 103.4 has 4 sig figs
 - 0 at the end of the number in the decimal places
 - Ex: 43.90 has 4 sig figs
 - 0 at the end of a number before a decimal point
 - Ex: 36030. Has 5 sig figs
 - Ex: 360 is ambiguous, but you would have to assume there are 2, because there is no decimal point
- Insignificant digits include:
 - 0 before any non-zero number
 - Ex: 0.005 has 1 significant digit (the 0s act as placeholders)
- Addition/Subtraction of Sig Figs
 - The answer should have the same number of **decimal** places as the number in the question with **the lowest** number of decimal points
 - Ex: $12.56 - 3.2 = 9.4$
 - It would have been 9.36, however, the number with the lowest number of decimal points has one decimal point (3.2), and you always round (up or down)
- Multiplication and Division
 - The answer should have the same number of **sig figs** as the number in the question with **the lowest** number of sig figs
 - Ex: $6.02 \times 9.0098 = 54.2$

- It would have been 54.238996, however the number with the lowest number of sig figs has 3 sig figs (6.02)
 - Sequential Calculations
 - If the calculation has both multiplication and division:
 - Follow BEDMAS, if addition/subtraction is done within brackets first, do not round, but keep track of how many decimal places should be used, then follow with multiplication/division. Between the number that was previously added/subtracted (with the correct number of decimal places) and the number to be divided, find the number with the lowest number of **sig figs** to use for the answer
 - Ex: $(1.2 + 3.04) \times 4.20 = 9.3$
 - $1.2 + 3.04 = 4.24$, however with the correct number of decimal places it would be 4.2
 - $4.24 \times 4.2 = 9.328$, but the correct number of sig figs is 1, because you are taking the number of sig figs of 4.2 instead of 4.24. Therefore the lowest number of sig figs is 1 sig fig.
 - The same is done in reverse if multiplication/division is done first

UNCERTAINTY

- Estimates all of the inaccuracies present in the measurement
 - Always expressed with one sig fig
- Absolute uncertainty
 - The size of the uncertainty using units
 - Ex: $\pm 2\text{cm}$
 - Meaning that the measurement can be 2cm greater or 2cm less than the actual measurement
 - Converting to Relative uncertainty
 - $(\text{Absolute Uncertainty}/\text{Measurement})$
 - Converting to Percentage uncertainty
 - $(\text{Absolute Uncertainty}/\text{Measurement}) \times 100$
 - Ex) $5.46 \pm 0.3\text{cm}$
 - $= 5.46 \pm ((0.3/5.46) \times 100) \text{ cm}$
 - $= 5.46 \pm 5.00\%$
- Relative/percentage uncertainty
 - Is the absolute uncertainty expressed as a fraction/percent of the measurement (therefore it has no units)
 - Ex: $\pm 2\%$
 - Meaning that the measurement can be 2% of the measurement greater or less than the actual measurement

- Converting to Absolute uncertainty
 - Change the percent value into decimal value (/100)
 - Multiply that value with the measurement
 - Ex) $3.21 \pm 1.2\%$
 - $= 3.21 \pm ((1.2/100) \times 3.21)$
 - $= 3.21 \pm 0.04$
- Addition/Subtraction
 - Add/subtract the measured values
 - Convert to absolute uncertainty
 - Add the absolute uncertainty values together
- Multiplication/Division
 - Multiply/divide the measured values
 - Convert to percentage uncertainty
 - Add the percentage values together
- Things to remember in labs
 - The uncertainty of the measurement can be found by taking half of the most uncertain digit
 - Ex) If you measure the mass of a solution as 13.6g, the uncertainty is half of the least certain digit (in this case it would be 6). Therefore, the uncertainty would be $13.6 \pm 0.3\text{g}$
 - The measured value and the uncertainty must have the same number of digits. Because uncertainty can only have one sig fig, add 0s to the end of the number
 - Ex) $5.46 \pm 0.3\text{cm}$
 - $= 5.46 \pm ((0.3/5.46) \times 100) \text{ cm}$
 - $= 5.46 \pm 5.00$
 - The answer calculated to 5.4945..., however uncertainty can only have 1 sig fig (in this case being 5), yet the measurement has 3 digits. Therefore the uncertainty works out to $5.46 \pm 5.00\text{cm}$