

Solubility and K_{sp}

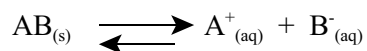
Solubility Calculations

Reminder: solubility is a concentration RATE.

Solubility is defined as **the maximum number of moles of solute that can be dissolved in one liter of solution at a given temperature.**

(NOTE: Mole of solution per liter of solution.)

When a solid ionic compound AB dissolves, the equation for equilibrium at saturation can be written as:



There is a **dynamic equilibrium** between undissolved solid and dissolved (aqueous) ions. The rate of dissolving (forward) is equal to the rate of crystallization (precipitation) (reverse).

The equilibrium constant for this equation system is called the **Solubility Product** or K_{sp} .

K_{sp} is the product of the concentrations of the ions in solution when the system is at saturation (equilibrium).

In the case above, $K_{sp} = [A^+][B^-]$

In general, where a compound X_mY_n reaches equilibrium with its aqueous solution as follows:



The solubility product (K_{sp}) is written as:

$$K_{sp} = [X^{n+}]^m [Y^{m-}]^n$$

The Table on page 5 gives some solubility product values at 25°C.

Example: the K_{sp} of lead chromate, $PbCrO_{4(s)}$, at 25 °C is 2.0×10^{-16}



$$[Pb^{2+}][CrO_4^{2-}] = 2.0 \times 10^{-16}$$

Therefore, $[Pb^{2+}] = [CrO_4^{2-}] = \sqrt{2.0 \times 10^{-16}} = 1.414 \times 10^{-8} \text{ mol/L}$

This means that lead chromate is insoluble. If one tries to dissolve lead chromate, the maximum amount that can be dissolved will be 1.4×10^{-8} moles. Small numbers like these mean that the compound is insoluble.

Use of K_{sp} to predict solubility

The solubility product, K_{sp} , is the product of the MAXIMUM AMOUNT of solute ions that can be dissolved at that temperature. The K_{sp} values are indicators of the solubility of compounds.

Generally speaking, small K_{sp} values mean low solubility (insoluble).

((Strictly speaking, ALL ionic compounds or polar covalent compounds are soluble to **some extent**.)

An arbitrary value is used for high school chemistry. **Below a solubility value of 0.1 mol/L, compounds are taken to be insoluble.**

Obviously, an “insoluble” lead or mercury or organometallic compound with a solubility of (say) 0.0001 mol/L would be hazardous to health if ingested.

Sample Problems

There are two types, using K_{sp} and finding K_{sp}

Example 1

The K_{sp} for Strontium chromate, $SrCrO_4$ is 3.60×10^{-5} . **Calculate the solubility** of $SrCrO_4$ at $25^\circ C$.

$$\begin{aligned} [Sr^{2+}][CrO_4^{2-}] &= 3.6 \times 10^{-5} \\ \text{i.e. } x^2 &= 36 \times 10^{-6} \\ \text{Therefore "x"} &= 6.0 \times 10^{-3} \text{ mol/L} \end{aligned}$$

This is the solubility of $SrCrO_4$

This calculation tells us that at $25^\circ C$ one cannot have higher concentration of $SrCrO_4$; i.e. Strontium chromate is insoluble for all practical purposes.

Generally, for simple AB compound, the solubility is the SQUARE ROOT of the K_{sp} and conversely, the K_{sp} is the SQUARE of the solubility.

$$\text{Sol} = \sqrt{K_{sp}}$$

$$K_{sp} = (\text{sol})^2$$

(NOTE: **only for AB compounds**, not A_2B , AB_2 , A_2B_3 etc.)

Example 2

A saturated solution of silver acetate, CH_3COOAg has a concentration of 0.05 mol/L. What is the K_{sp} ?

“AB” Type



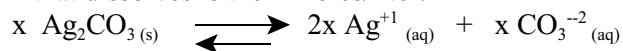
$$\begin{aligned} \text{i.e. } [Ag^+] &= [CH_3COO^-] = 5.0 \times 10^{-2} \text{ mol/L} \\ K_{sp} [Ag^+][CH_3COO^-] &= (5.2 \times 10^{-2})^2 \rightarrow \underline{2.5 \times 10^{-3}} \end{aligned}$$

Calculation for compound with the formula: “A₂B” typeExample 3

The K_{sp} for silver carbonate, Ag_2CO_3 is 6.2×10^{-12} . Calculate the solubility for silver carbonate.

Let the solubility of Ag_2CO_3 be “x” moles/L

i.e. the maximum that dissolves is the x moles/liter.



$$\begin{aligned} K_{sp} &= [Ag^+]^2[CO_3^{2-}] \\ &= (2x)^2(x) \\ &= 4x^3 \end{aligned}$$

$$\therefore 6.2 \times 10^{-12} = 4x^3$$

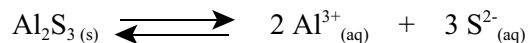
$$1.55 \times 10^{-12} = x^3$$

$$\therefore x = 1.16 \times 10^{-4} \text{ mol/L}$$

The solubility is 1.16×10^{-4} mol/L.

Calculation for compound with the formula: “A₂B₃” typeExample 4

The solubility of aluminum sulfide, Al_2S_3 is 2.0×10^{-7} mol/L. Calculate the K_{sp} for aluminum sulfide.



A concentration of 2.0×10^{-7} mol/L produces an **IONIC CONCENTRATION** as follows:

$$[Al^{3+}] = 4.0 \times 10^{-7} \text{ and } [S^{2-}] = 6.0 \times 10^{-7} \text{ mol/L}$$

The solubility product is:

i.e.

$$\begin{aligned} K_{sp} &= [Al^{3+}]^2[S^{2-}]^3 \\ K_{sp} &= (4.0 \times 10^{-7})^2 (6.0 \times 10^{-7})^3 \\ &= 3.5 \times 10^{-32} \end{aligned}$$

Use of K_{sp} to predict solubility

To check whether a precipitate will form when solutions are mixed, one calculates the **ion product**, (**trial product**) for ions in question in solution.

If the trial product is LESS THAN the K_{sp} , there will be no precipitate.

(The compound is soluble.)

If the trial product is GREATER THAN the K_{sp} there will be a precipitate.

(The ion concentrations are beyond solubility.)

Example: Given the K_{sp} for calcium sulfate so 2.4×10^{-5}

Case 1: 1.0 litre of 0.02 mol/L solution of CaCl_2 is mixed with 1.0 litre of 0.0004 mol/L solution of Na_2SO_4 .

Will a precipitate form? (A precipitate of CaSO_4)

Solution

Since the volume is doubled, the concentration of the reacting ions will be halved. ($C_1V_1 = C_2V_2$)

Therefore: $[\text{Ca}^{2+}] = 0.01 = 1.0 \times 10^{-2} \text{ mol/L}$
and $[\text{SO}_4^{2-}] = 0.0002 = 2.0 \times 10^{-4} \text{ mol/L}$.

Trial product: $[\text{Ca}^{2+}][\text{SO}_4^{2-}] = (1.0 \times 10^{-2})(2.0 \times 10^{-4})$
 $= 2.0 \times 10^{-6}$

This is less than the K_{sp} for calcium sulphate given in the question as 2.4×10^{-5}

Therefore, **precipitates will NOT occur.**

Case 2: Mix 1.0 litre of 0.08 mol/L CaCl_2 with 1.0 litre of 0.02 mol/L Na_2SO_4 . Will a precipitate of calcium sulphate form?

Solution

Since the volume is doubled, the concentration of the reacting ions will be halved. ($C_1V_1 = C_2V_2$)
(i.e. Divide conc. by 2)

Therefore: $[\text{Ca}^{2+}] = 4.0 \times 10^{-2} \text{ mol/L}$
and $[\text{SO}_4^{2-}] = 1.0 \times 10^{-2} \text{ mol/L}$

Trial product: $[\text{Ca}^{2+}][\text{SO}_4^{2-}] = (4.0 \times 10^{-2})(1.0 \times 10^{-2})$
 $= 4.0 \times 10^{-4}$

This is more than K_{sp} for calcium sulphate of 2.4×10^{-5} , therefore a precipitate will occur, until the excess ions are all precipitated. (Excess above solubility)

On the following pages are lists of common ions and a table of solubility.

Some K_{sp} ValuesIonic products and K_{sp} values for some substances at 25°C

SALT	IONIC PRODUCT	K_{sp}
AgC ₂ H ₃ O ₂	[Ag ⁺][C ₂ H ₃ O ₂ ⁻]	2.5 x 10 ⁻³
AgBr	[Ag ⁺][Br ⁻]	4.8 x 10 ⁻¹³
Ag ₂ CO ₃	[Ag ⁺] ² [CO ₃ ⁼]	8.2 x 10 ⁻¹²
AgCl	[Ag ⁺][Cl ⁻]	1.2 x 10 ⁻¹⁰
AgI	[Ag ⁺][I ⁻]	1.5 x 10 ⁻¹⁶
Ag ₂ S	[Ag ⁺] ² [S ⁼]	1.6 x 10 ⁻⁴⁹
Al(OH) ₃	[Al ³⁺][OH ⁻] ³	5.0 x 10 ⁻³³
BaCO ₃	[Ba ⁺⁺][CO ₃ ⁼]	4.9 x 10 ⁻⁹
BaSO ₄	[Ba ⁺⁺][SO ₄ ⁼]	1.1 x 10 ⁻¹⁰
CdS	[Cd ⁺⁺][S ⁼]	1.0 x 10 ⁻²⁸
CaCO ₃	[Ca ⁺⁺][CO ₃ ⁼]	4.8 x 10 ⁻⁹
CaF ₂	[Ca ⁺⁺][F ⁻] ²	4.2 x 10 ⁻¹¹
Ca(OH) ₂	[Ca ⁺⁺][OH ⁻] ²	1.3 x 10 ⁻⁶
CaSO ₄	[Ca ⁺⁺][SO ₄ ⁼]	6.1 x 10 ⁻⁵
CuCl	[Cu ⁺][Cl ⁻]	3.2 x 10 ⁻⁷
Cu ₂ S	[Cu ⁺] ² [S ⁼]	1.6 x 10 ⁻⁴⁸
CuS	[Cu ⁺⁺][S ⁼]	8.5 x 10 ⁻⁴⁵
FeS	[Fe ⁺⁺][S ⁼]	3.7 x 10 ⁻¹⁹
Fe(OH) ₃	[Fe ³⁺][OH ⁻] ³	1.5 x 10 ⁻³⁶
HgS	[Hg ⁺⁺][S ⁼]	3.0 x 10 ⁻⁵³
MgCO ₃	[Mg ⁺⁺][CO ₃ ⁼]	2.5 x 10 ⁻⁵
Mg(OH) ₂	[Mg ⁺⁺][OH ⁻] ²	5.9 x 10 ⁻¹²
MnS	[Mn ⁺⁺][S ⁼]	1.4 x 10 ⁻¹⁵
NiS	[Ni ⁺⁺][S ⁼]	1.8 x 10 ⁻²¹
PbCl ₂	[Pb ⁺⁺][Cl ⁻] ²	1.0 x 10 ⁻⁴
PbCrO ₄	[Pb ⁺⁺][CrO ₄ ⁼]	1.8 x 10 ⁻¹⁴
PbSO ₄	[Pb ⁺⁺][SO ₄ ⁼]	1.9 x 10 ⁻⁸
PbS	[Pb ⁺⁺][S ⁼]	8.4 x 10 ⁻²⁸
SrSO ₄	[Sr ⁺⁺][SO ₄ ⁼]	2.8 x 10 ⁻⁷
ZnS	[Zn ⁺⁺][S ⁼]	4.5 x 10 ⁻²⁴

Solubility of Common Inorganic Compounds in Water
(Low solubility = "insoluble")

Negative Ions (anions)	+	Positive Ions (cations)	-	Compounds with the solubility
Essentially all		Alkali ions (Li ⁺ , Na ⁺ , K ⁺ , Rb ⁺ , Cs ⁺ , Fr ⁺)		Soluble
Essentially all		Hydrogen ions [H ⁺ _(aq)]		Soluble (ACIDS)
Essentially all		Ammonium ions (NH ₄ ⁺)		Soluble
Nitrate, NO ₃ ⁻		Essentially all		Soluble
Acetate, CH ₃ COO ⁻		Essentially all		Soluble
Chloride, Cl ⁻		Ag ⁺ , Pb ⁺ , Hg ²⁺ , Cu ⁺ ,		Low solubility
Bromide, Br ⁻		Ti ⁺		
Iodide, I ⁻		All others		Soluble
Sulfate, SO ₄ ²⁻		Ca ²⁺ , Sr ²⁺ , Ba ²⁺ , Pb ²⁺ , Ra ²⁺		Low solubility
		All others		Soluble
Sulfide, S ₂ ⁻		Alkali ions, H ⁺ _(aq) , NH ₄ ⁺ , Be ²⁺ , Mg ²⁺ , Ca ²⁺ , Sr ²⁺ , Ba ²⁺ , Ra ²⁺		Soluble
		All others		Low solubility
Hydroxide, OH ⁻		Alkali ions, H ⁺ _(aq) , NH ₄ ⁺ , Sr ²⁺ , Ba ²⁺ , Ra ²⁺ , Ti ⁺		Soluble
		All others		Low solubility
Phosphate, PO ₄ ³⁻		Alkali ions, H ⁺ _(aq) ,		Soluble
Carbonate, CO ₃ ²⁻		NH ₄ ⁺		
Sulfite, SO ₃ ²⁻		All others		Low solubility