Measurement and data processing

2007 - 2008

Reference: IB Topic 11

One of the major components of data processing involves the comparison of the experimental value with the theoretical or the literature value.

No measurement is absolute: all measurements have associated uncertainty, generally in the last significant figure, since the actual value lies in a range above and below the reading . The last digit recorded in the measurement represent a guess or the uncertainty in the measurement.
The uncertainty often referred to as an "error", is not a result of a mistake.
The uncertainty is associated with a measurement is due to the equipment being used to perform the experiment, e.g. a balance or a burette or a pipette.

Measurement errors can be classified as <u>systematic</u> or <u>random.</u>

Systematic Errors (a.k.a.: bias error)

A systematic error is one which can be assigned to a definite cause and can be usually eliminated.

For example, absorption of water by a hygroscopic (fyi: hygroscopic refers to compounds that remove moisture from the air) substance would cause its measured mass to be greater than the true mass.
Such an error will result in a loss of accuracy, (accuracy is a means of expressing the difference between an observed value and the 'correct' or 'true' value).
The effect of such an error on the precision (i.e. the reproducibility, or the extent of variation among multiple measurements), depends on whether the error is variable or constant.

Systematic errors are due to incorrect use of equipment or poor experimental design.

<u>There are four types of systematic errors:</u>
1. Instrumental - calibration error

2.Consistent bias in observation, e.g. when reading a burette

3. Natural or environmental errors, e.g. external influences such as temperature and atmospheric pressure affect the measurement of gas volumes.

4. Theoretical, (due to approximations or simplifications made, e.g. assuming no heat losses in calorimetry measurements).

In the most accurate work, equipment such as burettes and balances must be calibrated to correct for systematic errors.

Repeating trials will not reduce systematic error.

Systematic error acts to impair the accuracy of the measurement but has no effect on precision

A systematic error will make the measured value either smaller or larger than the actual value., but not both simultaneously.

Random Errors

A random error is due to the effects of uncontrolled variables and exhibits no pattern; it can make the measured value both smaller and larger than the true value but tend towards the true value as the number of repeat measurements increases.

Random error results from small variations in measurements due to randomly changing conditions, e.g. weather, humidity, quality of equipment, etc.

Random error is unbiased.

Uncertainties are measures of random error, generally they can be assumed to be half of the smallest division of the scale.

Random error will decrease the reproducibility of the measurements, i.e. a random error will affect precision.

Statistical analysis may be applied to random errors in multiple measurements of the same quantity.

Error analysis in chemistry

Error exists in every measured or experimentally obtained value.

(Errors are stated to only one significant figure.)

The experimental uncertainty is the best estimate of the total of all inaccuracies present in that measurement.

<u>The uncertainty can be expressed in two different ways:</u>

1. Absolute uncertainty: the size of the uncertainty is expressed using units with no indication of its importance.
   e.g. $\pm$ 0.01 g $\pm$ 0.05 cm $\pm$ 0.001 mL

2. Relative uncertainty: the absolute uncertainty is expressed as a fraction or as a percent of the measured value, and has no units.

$$\textbf{Relative uncertainty} = \frac{\textbf{absolute uncertainty}}{\textbf{measured value}}$$

$$\textbf{percentage uncertainty} = \textbf{relative uncertainty} \times \textbf{100 \%}$$

Example: 3.51 $\pm$ 0. 2 g

absolute uncertainty = 0.2 g

$$\text{relative uncertainty} = \frac{0.2\text{ g}}{3.51\text{ g}} = 0.06$$

percentage uncertainty = 0.06 x 100 = 6 %

Performing calculations with measured values and their uncertainties

For Addition and / or Subtraction

1. Simply add or subtract the measured values as described by the operation sign.
2. Then add together the sizes of all the absolute uncertainties in each measurement to find the total uncertainty.

e.g. (i) (3.51 ± 0.001 g) + (4.03 ± 0.001 g)
= 7.54 ± 0.002 g

e.g. (ii) 55.5 ± 0.10 ^{O}C — 25.5 ± 0.10 ^{O}C
= 30.0 ± 0.20 ^{O}C

For Multiplication and /or Division

1. Multiply and /or divide the measured values as described by the operation sign.
2. Add together the sizes of all the relative or the % relative uncertainties for each measurements being multiplied/divided..
3. The absolute error is then the fraction or the percentage of the answer.

Example 1:

Mass of a cube = 13.252 ± 0.002 g
volume of cube = 6.25 mL ± 0.10 mL
Calculate the density of the cube.

Solution:

1. Density = $\frac{\text{Mass}}{\text{Volume}} = \frac{13.252 \text{ g}}{6.25 \text{ mL}}$ = 2.12 (032) g ml^{-1}

2. Calculate the relative uncertainty for the mass and the volume of the cube:

relative uncertainty for mass = $\frac{0.002}{13.252}$ = 1.51 x10^{-4}

relative uncertainty for volume = $\frac{0.10}{6.25}$ = 0.016

sum of relative uncertainties = 0.0162

3. Convert relative uncertainty back to absolute uncertainty:
2.12 x 0.0162 = 0.03

Hence, the density of the cube = 2.12 ± 0.03 g mL^{-1}

Example 2: What is the product of 3.6 ± 0.5cm and 2.6 ± 0.5 cm

Solution:

1. Determine the product of 3.6 cm and 2.6 cm = 9.36 cm^2
2. Relative uncertainty for the measurement 3.6 = $\frac{0.5 \text{ cm}}{3.6 \text{ cm}}$ = 0.139

Relative uncertainty for the measurement 2.6 = $\frac{0.5 \text{ cm}}{2.6 \text{ cm}}$ = 0.192

Sum of relative uncertainties = 0.331

3. Absolute error = 9.36 cm^2 x 0.331 = 3.10 cm^2

Hence the product of 3.6 ± 0.5cm and 2.6 ± 0.5 cm = 9.36 ± 3 cm^2

Multiplying or dividing by a pure number (i.e. with no uncertainty)

Multiply or divide the absolute uncertainties by the pure number.
E.g. (2.3 ± 0.10) x 2.0 = 4.6 ± 0.20

Practice:

1. A sample of aluminium is found to have a mass of 11.26 $\pm$ 0.05 g and a volume of 4.31 mL $\pm$ 0.1 mL.

a. What are the relative uncertainties in the mass and the volume. (Answer: 0.4 %, 2 %)

b. What is the experimental value for the density of the aluminium? (d = 2.61 $\pm$ 0.07 g mL^{-1})

c. What are the relative and absolute uncertainties in the experimental density? ($\pm$ 0.07 g mL^{-1}, 2%)

d. Given that the density of aluminium is 2.71 g mL^{-1} , what is the experimental error? (3.69%)

2. A piece of aluminium foil has a length of 12.20 $\pm$ 0.01 cm, width of 10.35 $\pm$ 0.01 cm, and a mass of 0.85 $\pm$ 0.01 g. Given that the density of the aluminium foil is 2.71 g cm^{-3}, in order to determine the thickness of the aluminium foil follow the following operations for the calculation:

a. The volume of the aluminium foil (v = m / d)

b. The relative uncertainty in the mass and the volume.

c. The volume of the aluminium foil, stating the absolute uncertainty.

d. The area of the aluminium foil (area = l x w)

e. The relative uncertainty in the length and the width.

f. The area of the aluminium foil, stating the absolute uncertainty.

g. The thickness of the aluminium foil (T = volume / area)

h. The relative uncertainty of the volume and the area.

i. State the thickness of the aluminium foil with the absolute uncertainty.